EVAPORATION OF ARBITRARY-SIZE DROP IN ELECTROMAGNETIC RADIATION FIELD

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A theory of the evaporation of an arbitrary-size drop in a field of electromagnetic radiation is constructed using the Lees method. An analytic expression is found for the heat and mass fluxes. An equation describing the change in drop radius as a function of time is obtained.

Consider a pure drop of volatile material (radius R) in a binary gas mixture, the first component of which consists of molecules of the volatile drop material and the second of inert-gas molecules.

Monochromatic electromagnetic radiation of wavelength λ_0 falls on the drop. Part of the radiation energy is absorbed by the drop and consumed in evaporation. Following [1], it will be assumed that the thermal energy released in the drop volume is homogeneously distributed in it with quantity q

$$q = \frac{3IK_p(R,\ \lambda_0)}{4R}, \tag{1}$$

where I is the radiation intensity; K_p is the absorption coefficient [1].

To determine the evaporation time of the drop it is necessary to know the explicit form of the expressions for the heat-flux density released from the drop surface Q and the material of the first (volatile) component N_1. The values of Q and N_1 will be found in a quasistationary approximation by the Lees method from the solution of the kinetic Boltzmann equation under the condition that $n_1 \ll n$ and $(T_s - T_\infty)/T_\infty \ll 1$ (n_1 and n_2 are the molecular densities of the first and second components of the gas mixture; $n = n_1 + n_2$; T_s is the temperature of the drop surface; and T_∞ is the gas temperature at infinity).

The condition $n_1 \ll n$ is necessary since the Lees method does not take into account the convective motion of the medium and therefore is only used for $n_1 \ll n$. In its turn, this condition significantly simplifies the derivation of the expression for N_1.

The use of the Lees method for drop evaporation is outlined in detail in [6, 7]. Therefore, the expressions for the flux densities N_1 and Q are not derived here but simply stated as (2), (3):

TABLE 1. Surface Temperature of Water Drop as a Function of Drop Radius R and Intensity of Electromagnetic Radiation I, W/cm²

R, μ	T_s, °K		
	I = 10	I = 10^2	I = 10^3
10^2	330	—	—
10	297	317	—
1	294,5	296	308
10^{-1}	294	294,5	295,5
10^{-2}	293,5	294	295
10^{-3}	293,2	293,5	294

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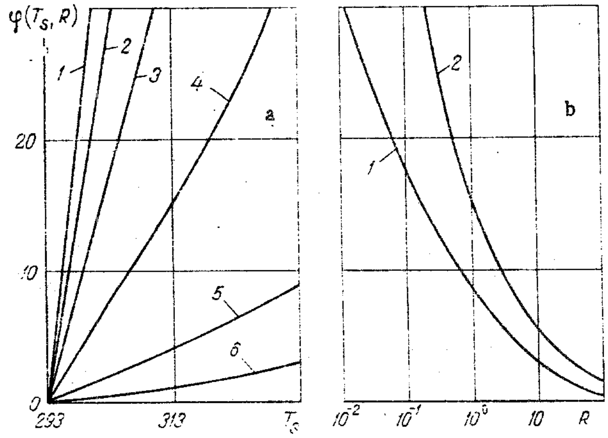

Fig. 1. Values of $\varphi(T_s, R)$, cal/cm^2 · sec for fixed values of the radius (a) and fixed temperature (b): a) $R = 10^{-3}$ μ(1), 10^{-2} (2), 10^{-1} (3), 1 (4), 10 (5), 10^2 μ (6); b) T_s = 303 (1), 313°K (2).

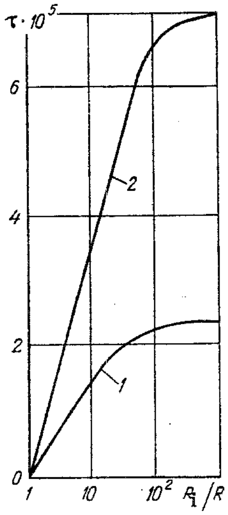

Fig. 2

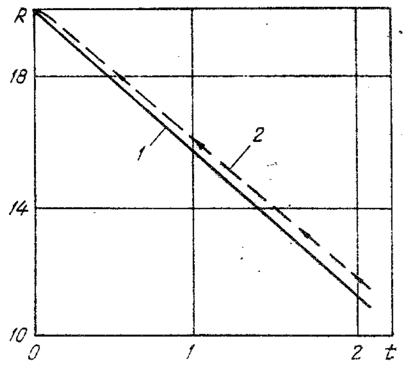

Fig. 3

Fig. 2. Dependence of drop radius on dimensionless evaporation time τ.

Fig. 3. Comparison of the experimental and theoretical dependence of R, μ, on t, sec.

$$N_1 = \sqrt{\frac{2kT_\infty}{\pi m_1}}\,\alpha_{m_1}\left(1+\frac{n_{1\infty}}{n_{2\infty}}\right)\frac{\exp\left(\frac{2V_1\delta}{RkT_s}\right)n_{1s}-n_{1\infty}}{1+\alpha R/\lambda}\,\frac{R^2}{r^2}, \tag{2}$$

$$\dot{Q} = \sqrt{\frac{2kT_\infty}{\pi m_2}}\,n_{2\infty}\,\frac{T_s-T_\infty}{1+\beta R/\lambda}\,\frac{R^2}{r^2}, \tag{3}$$

where $n_{1\infty}$ and $n_{2\infty}$ are the molecular densities of the vapor and inert gas at infinity; m_1 and m_2 are the molecular masses of the vapor and gas; α_{m_1} is the evaporation factor of the volatile component; n_{1s} is the saturation vapor density at temperature T_s; $\exp(2V_1\delta/RkT_s)$ is the Kelvin factor; k is the Boltzmann constant; λ is the mean free path of the gas-mixture molecules; α and β are coefficients

$$\alpha = \alpha_{m_1}\frac{2\sqrt{2}}{3\pi}\left(1+\frac{\sigma_1}{\sigma_2}\right)^2\left(1+\frac{m_1}{m_2}\right)^{-1/2},$$

$$\beta = \frac{32}{15\pi}$$

(σ_1 and σ_2 are the diameters of the vapor and gas molecules).

In deriving Eqs. (2)-(3) the model collision integrals in Gross and Krook form were used [8]. Equation (2) is considerably simpler in structure than the formula for the vapor flux obtained in [6]. In the limiting cases for $R \gg \lambda$ and $R \ll \lambda$, Eq. (2) transforms to the Shankar formula. These expressions give similar values for vapor fluxes in the intermediate range of Knudsen $Kn = \lambda/R$, i.e., for $R \sim \lambda$. For example, in the case of air–vapor mixture the maximum difference between the vapor fluxes calculated from Eq. (2) and the Shankar formula does not exceed 30%.

The heat transfer is described using an expression obtained for a one-component ($n_1 \ll n_2$) monoatomic gas. Applications usually involve monoatomic gases. As comparison with experiment shows, the estimate is expediently made using Eq. (4), which takes into account the internal degrees of freedom of the gas molecules

$$Q = \sqrt{\frac{2kT_\infty}{\pi m_2}}\, n_{2\infty} f \frac{T_s - T_\infty}{1 + \beta R/\lambda} \frac{R^2}{r^2}, \tag{4}$$

where f is the Einstein correction factor [10]

$$f = 1.3 + \frac{3.6}{c_v} - \frac{0.92 c_v - 2.75}{(z + 0.53 c_v - 0.76) c_v}$$

(c_V is the molar specific heat of the gas at constant volume; z is the correction for inelastic collision).

The distribution of the temperature T_1 in the drop volume is described by the Laplace equation, the solution of which is

$$T_i = T_s + \frac{IK_p R}{8\varkappa_i}\left(1 - \frac{r^2}{R^2}\right), \tag{5}$$

where $\varkappa_1$ is the thermal conductivity of the drop material.

In Eqs. (2)-(5) the unknowns are the saturated vapor density and the temperature T_S, which may be found from Eq. (7); this equation is obtained by substituting Eqs. (2), (4), and (5) into Eq. (6), taking into account the continuity of the heat flux passing through the drop surface

$$Q(R) + Lm_1 N_1(R) = -\varkappa_i \left.\frac{dT_i}{dr}\right|_{r=R}, \tag{6}$$

$$\varphi(T_s, R) \equiv \sqrt{\frac{2kT_\infty}{\pi m_1}}\left\{n_{2\infty} fk \sqrt{\frac{m_1}{m_2}} \frac{T_s - T_\infty}{1 + \beta R/\lambda} + L\alpha_{m_1} m_1 \left(1 + \frac{n_{1\infty}}{n_{2\infty}}\right) \frac{\exp\left(\frac{2V_1\delta}{RkT_s}\right) n_{1s} - n_{1\infty}}{1 + \alpha R/\lambda}\right\} = \frac{IK_p}{4}, \tag{7}$$

where L is the heat of phase transition.

In the general case Eq. (7) is transcendental and cannot be solved analytically. The dependence of $\varphi(T_S, R)$ on T_S for a water drop is shown in Fig. 1a for fixed values of R; from these curves, knowing $IK_p/4$ it is simple to determine T_S and hence n_{1S}. The dependence of $\varphi(T_S, R)$ on R for fixed values of T_S is shown in Fig. 1b (R is plotted in a logarithmic scale along the abscissa).

Table 1 shows values of T_S obtained using the curves in Fig. 1a for various values of I and R. As is evident from Table 1, for a drop radius $R \leq 10^{-1}$ μ even at an intensity $I = 10^3$ W/cm^2 the surface temperature of the drop differs only slightly from the temperature of the unperturbed vapor–gas mixture (in obtaining the values of T_S it is assumed that in the absence of radiation the drop is in thermodynamic equilibrium with the surrounding medium and $T_\infty = 293$°K).

The dependence of the drop radius on the time t is described by the conservation equation for the drop mass M

$$-\frac{dM}{dt} = 4\pi R^2 m_1 N_1(R), \tag{8}$$

where N_1 may be found from Eq. (2) using Eq. (7).

In the general case Eq. (8) does not integrate in quadratures and it must be solved numerically.

From Eqs. (2), (7), and (8) the dependence of the drop radius R on the evaporation time t was estimated for a water drop evaporating in air under the action of CO_2-laser radiation of wavelength $\lambda_0 = 0.16\ \mu$ and intensity $I = 3 \cdot 10^2$ and 10^3 W/cm^2. The results are shown in Fig. 2: Curve 1 is plotted for a water drop of initial radius $R_i = 3\ \mu$, evaporating in a radiation field of intensity $I = 3 \cdot 10^2$ W/cm^2 and curve 2 corresponds to $R_i = 1\ \mu$ and $I = 10^3$ W/cm^2. The ratio R_i/R is plotted in a logarithmic scale along the abscissa and the dimensionless time $\tau = tD_{12}/R_i^2$ is plotted along the ordinate (D_{12} is the binary-diffusion coefficient). As is evident from Fig. 1, when a water drop of radius less than the wavelength of the electromagnetic radiation evaporates ($R < \lambda_0$), $\tau \sim \ln(R_i/R)$ in fields of sufficiently large intensity in the first stage. This dependence arises because the main mechanism of heat liberation from the drop initially is evaporation of the drop material since $Q \ll Lm_1N_1$ and therefore the flux density $N_1(R) \sim K_p$. In turn, the absorption factor $K_p \sim R$.

With decrease in drop size in the course of evaporation its surface temperature falls and molecular heat conduction plays a more important part in the heat-liberation mechanism (this period corresponds to the elbow in the curves). Finally, when the radius becomes very small ($R \ll \lambda$) evaporation occurs mainly as a result of the effect of the Kelvin factor, since heat liberation in the volume of the drop as a result of absorption of electromagnetic energy is very small at this drop size and the concentration $n_{1\infty}$ is assumed in the calculation to be equal to the saturation vapor concentration at temperature T_∞ (this period corresponds to the section of slow variation on the curves).

In Fig. 3 the theoretical results are compared with the experimental data of [8] obtained in the evaporation of a water drop in a field of electromagnetic radiation of intensity $I = 25$ W/cm^2 ($\lambda_0 = 10.6\ \mu$): Curve 1 is plotted from Eqs. (7)-(8) and curve 2 is taken from [9]. The agreement between theory and experiment is evidently good. The dependence of R on t in this figure is linear because the heat liberation in the considered range of drop radii occurs mainly by evaporation of the drop material and the absorption factor K_p (in contrast to the case in Fig. 2) is essentially unchanged in this range of drop-size variation.

Unfortunately, the literature does not include any experimental data on evaporation of drops with large Knudsen numbers in the field of electromagnetic radiation.

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